| 1 | (i) | $\begin{aligned} f(-x) & =\frac{2(-x)}{1-(-x)^{2}} \\ & =-\frac{2 x}{1-x^{2}}=-\mathrm{f}(x) \end{aligned}$ | M1 <br> A1 <br> [2] | substitutin $-x$ for $x$ in $\mathrm{f}(x)$ |
| :---: | :---: | :---: | :---: | :---: |
|  | (ii) |  | M1 <br> A1 <br> [2] | Recognisable attempt at a half turn rotation about O <br> Good curve starting from $x=-4$, asymptote $x=-1$ shown on graph. (Need not state -4 and -1 explicitly as long as graph is reasonably to scale.) Condone if curve starts to the left of $x=-4$. |


| 2 |  | $\operatorname{fg}(x)=\ln \left(1+x^{2}\right)$ | $(x \in \mathfrak{R})$ | B1 | condone missing bracket, and | If fg and gf the wrong way round, B1B0 |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\operatorname{gf}(x)=1+(\ln x)^{2}$ | $(x>0)$ | B1 | missing or incorrect domains | not $1+\ln \left(x^{2}\right)$ |  |  |  |
|  |  |  |  | B1 | Penalise missing bracket |  |  |
|  |  |  | B1 $1+(\ln x)^{2}$ neither |  | Penalise missing bracket |  |  |
|  |  |  |  |  |  |  |  |


| 3 | (i) | (One-way) stretch in $y$-direction, s.f. 2 or in $x$-direction s.f. $1 / 2$ translation 1 to right (2 if followed by $x$-stretch) $y=2\|x-1\|$ | B1 <br> B1 <br> B1 <br> [3] | must specify s.f. and direction <br> o.e. e.g. $y=\|2 x-2\| y=\|2(x-1)\|$ | Allow 'compress', ‘squeeze'(for s.f. $1 / 2$ ), but not 'enlarge', ' $x$-coordinates halved', etc Allow 'shift','move' or vector only, 'right 1' Don't allow misreads (e.g. transforming solid graph to dashed graph) <br> Award B1 for one of these seen, and a second B1 if combined transformations are correct |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (ii) | Reflection in $x$-axis or translation right $\pm \pi$ or rotation of $180^{\circ}$ [about O] translation +1 in $y$-direction ( -1 if followed by reflection in $x$-axis $y=1-\cos x$ | B1 B1 <br> B1 <br> [3] | $\binom{ \pm \pi}{1} \text { is B2 }$ <br> allow $1+\cos (x \pm \pi)$ (bracket needed) | Translations as above. <br> Reflection: must specify axis, allow 'flip' <br> Rotation: condone no origin stated. <br> See additional notes for other possible solutions. <br> Award B1 for any one of these seen, and a second B1 if combined transformations are correct |


| Question |  | Answer | Marks | Guidance |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 4 | (i) | $\begin{aligned} & 1-9 a^{2}=0 \\ & \Rightarrow a^{2}=1 / 9 \Rightarrow a=1 / 3 \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \\ & \text { [2] } \end{aligned}$ | $\begin{aligned} & \text { or } 1-9 x^{2}=0 \\ & \text { or } 0.33 \text { or better } \sqrt{ }(1 / 9) \text { is A0 } \end{aligned}$ | $\begin{aligned} & \sqrt{ }\left(1-9 a^{2}\right)=1-3 a \text { is M0 } \\ & \text { not } a= \pm 1 / 3 \text { nor } x=1 / 3 \end{aligned}$ |
| 4 | (ii) | Range $0 \leq y \leq 1$ | $\begin{aligned} & \text { B1 } \\ & \text { [1] } \end{aligned}$ | $\begin{aligned} & \text { or } 0 \leq \mathrm{f}(x) \leq 1 \text { or } 0 \leq \mathrm{f} \leq 1, \text { not } 0 \leq x \leq 1 \\ & 0 \leq y \leq \sqrt{ } 1 \text { is } \mathrm{B} 0 \end{aligned}$ | allow also [0,1], or 0 to 1 inclusive, but not 0 to 1 or ( 0,1 ) |
| 4 | (iii) |  | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \\ & \text { A1 } \\ & \text { [3] } \\ & \hline \end{aligned}$ | curve goes from $x=-3 a$ to $x=3 a$ (or -1 to 1 ) <br> vertex at origin curve, 'centre’ $(0,-1)$, from $(-1,-1)$ to $(1,-1)$ ( $y$-coords of -1 can be inferred from vertex at O and correct scaling) | must have evidence of using s.f. 3 allow also if s.f. 3 is stated and stretch is reasonably to scale <br> allow from $(-3 a,-1)$ to $(3 a,-1)$ <br> provided $a=1 / 3$ or $x=[ \pm] 1 / 3$ in (i) <br> A0 for badly inconsistent scale(s) |


| 5 | (i) | $\begin{aligned} \mathrm{s}(-x) & =\mathrm{f}(-x)+\mathrm{g}(-x) \\ & =-\mathrm{f}(x)+-\mathrm{g}(x) \\ & =-(\mathrm{f}(x)+\mathrm{g}(x)) \\ & =-\mathrm{s}(x) \quad(\text { so s is odd }) \end{aligned}$ | M1 <br> A1 <br> [2] | must have $s(-x)=\ldots$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | (ii) | $\begin{aligned} & \mathrm{p}(-x)=\mathrm{f}(-x) \mathrm{g}(-x) \\ &=(-\mathrm{f}(x)) \times(-\mathrm{g}(x)) \\ &=\mathrm{f}(x) \mathrm{g}(x)=\mathrm{p}(x) \\ & \text { so } \mathrm{p} \text { is even } \end{aligned}$ | M1 <br> A1 <br> [2] | must have $\mathrm{p}(-x)=\ldots$ <br> Allow SC1 for showing that $\mathrm{p}(-x)=\mathrm{p}(x)$ using two specific odd functions, but in this case they must still show that $p$ is even | e.g. $\mathrm{f}(x)=x, \mathrm{~g}(x)=x^{3}, \mathrm{p}(x)=x^{4}$ $p(-x)=(-x)^{4}=x^{4}=p(x)$, so $p$ even condone $f$ and $g$ being the same function |


| 6 (i)f(-x) $=\mathrm{f}(x)$ <br> Symmetrical about Oy. | B1 |  |  |
| :--- | :--- | :--- | :--- |
|  |  | B1 |  |
|  | [2] |  |  |
| (ii)(A) even <br>  <br>  <br>  <br>  <br> (B) nei her <br> (C) od | B1 |  |  |
|  | B1 |  |  |


| 7 (i) (A) <br> (B) | B1 <br> B1 <br> M1 <br> A1 <br> [4] | Zeros shown every $\pi / 2$. <br> Correct shape, from $-\pi$ to $\pi$ <br> Translated in $x$-direction <br> $\pi$ to the left |
| :---: | :---: | :---: |


| $\text { (ii) } \begin{aligned} & \mathrm{f}^{\prime}(x)=-\frac{1}{5} e^{-\frac{1}{5} x} \sin x+e^{-\frac{1}{5} x} \cos x \\ & \mathrm{f}^{\prime}(x)=0 \text { when }-\frac{1}{5} e^{-\frac{1}{5} x} \sin x+e^{-\frac{1}{5} x} \cos x=0 \\ & \Rightarrow \frac{1}{5} e^{-\frac{1}{5} x}(-\sin x+5 \cos x)=0 \\ & \Rightarrow \sin x=5 \cos x \\ & \Rightarrow \frac{\sin x}{\cos x}=5 \\ & \Rightarrow \tan x=5^{*} \\ & \Rightarrow x=1.37(34 \ldots) \\ & \Rightarrow y=0.75 \text { or } 0.74(5 \ldots) \end{aligned}$ | B1 <br> B1 <br> M1 <br> E1 <br> B1 <br> B1 <br> [6] | $\begin{aligned} & e^{-\frac{1}{5} x} \cos x \\ & \ldots-\frac{1}{5} e^{-\frac{1}{5} x} \sin x \end{aligned}$ <br> dividing by $e^{-\frac{1}{5} x}$ <br> www <br> 1.4 or better, must be in radians 0.75 or better |
| :---: | :---: | :---: |
| (iii) $\begin{aligned} & \mathrm{f}(x+\pi)=e^{-\frac{1}{5}(x+\pi)} \sin (x+\pi) \\ & =e^{-\frac{1}{5} x} e^{-\frac{1}{5} \pi} \sin (x+\pi) \\ & =-e^{-\frac{1}{5} x} e^{-\frac{1}{5} \pi} \sin x \\ & =-e^{-\frac{1}{5} \pi} \mathrm{f}(x)^{*} \\ & \int_{\pi}^{2 \pi} \mathrm{f}(x) d x \text { let } u=x-\pi, \mathrm{d} u=\mathrm{d} x \\ & =\int_{0}^{\pi} \mathrm{f}(u+\pi) d u \\ & =\int_{0}^{\pi}-e^{-\frac{\pi}{5} \pi} \mathrm{f}(u) d u \\ & =-e^{-\frac{1}{5} \pi} \int_{0}^{\pi} \mathrm{f}(u) d u^{*} \end{aligned}$ <br> Area enclosed between $\pi$ and $2 \pi$ $=(-) e^{-\frac{1}{5} \pi} \times$ area between 0 and $\pi$. | M1 <br> A1 <br> A1 <br> E1 <br> B1 <br> B1dep <br> E1 <br> B1 <br> [8] | $\begin{aligned} & e^{-\frac{1}{5}(x+\pi)}=e^{-\frac{1}{5} x} \cdot e^{-\frac{1}{5} \pi} \\ & \sin (x+\pi)=-\sin x \\ & \mathrm{www} \end{aligned}$ <br> $\int f(u+\pi) d u$ <br> limits changed <br> using above result or repeating work <br> or multiplied by 0.53 or better |

